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Unsteady conjugate laminar heat transfer of a rotating non-uniformly heated disk: Application to the transient experimental technique

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Abstract

Presented in the paper are the results of an investigation of 2D heat conduction effects on the transient heat transfer of a rotating disk heated up to a non-uniform initial temperature and suddenly subjected to unsteady cooling by still air. A self-similar solution of the transient laminar convective heat transfer confirmed that the heat transfer coefficient rapidly becomes time-independent and equal to its value at steady-state conditions. An analytical solution of the unsteady two-dimensional heat conduction inside a disk made of Plexiglas® confirmed that the known infinite-slab approach can still be used as a transient technique for determining heat transfer coefficients. Use of the regular heat transfer regime theory for the same purpose can be recommended only for the cases with the moderate initial temperature non-uniformity.

Keywords: Heat transfer; Rotating; Transient

1. Introduction

Transient heat transfer investigations are a matter of great importance both from the fundamental and applied points of view. One of the most widely used applications comprise various transient experimental techniques of determining surface heat transfer of a body. Such techniques, currently employing thermochromic liquid crystals, are based on the known fact that after a certain period of time from the beginning of the cooling process, the surface heat transfer coefficient becomes a time-independent function equal to its value for steady-state heat transfer under the same boundary conditions. The heat flux in this situation can be computed from more or less simple analytical solutions for unsteady heat conduction inside solid bodies at known surface temperatures [1].

Most often, the experimental data analysis is based on the theory of one-dimensional heat conduction in a semi-infinite slab with a convective boundary condition at the interface between the slab and the cooling/heating medium with a step change in the coolant's temperature T_∞ [2–5]

$$F_t(t) = \frac{T_{\rm w}(t) - T_\infty}{T_{{\rm w},i} - T_\infty} = \exp(\gamma^2) \cdot \operatorname{erfc}(\gamma),$$
$$\gamma = h_2\sqrt{a_{\rm w}t}/k_{\rm w}. \qquad (1)$$

Having measured the temporal curve of the local surface temperature, one can solve (1) for the heat transfer coefficient. Use of this technique is restricted by the obvious consideration that heat conduction must involve only a small fraction of the wall thickness for the infinite-slab assumption to hold. Therefore, the measurement time is strictly limited: by the value $Fo = 1/4$ in accordance with the classical theory [6] or $Fo = 1$ according to [7]. As shown in [8,9], significant deviations of the experimental results [3] from the standard data of other authors might have occurred because of disregarding the aforementioned restriction in the measurement time.

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Nomenclature

a	thermal diffusivity
b	outer radius of disk
Bi_1	Biot number $h_1 b/k_w$
Bi_2	Biot number $0.5 h_2 s/k_w$
c_{0*}	constant in Eq. (4) i.e. $\Delta T_{n_*=0}$
$F_t(t)$	relative instantaneous wall temperature, $(T_w(t,r) - T_\infty)/(T_{w,i}(r) - T_\infty)$
Fo	Fourier number $4a_w t/s^2$
h	heat transfer coefficient
H	parameter, $b/(s/2)$
k	thermal conductivity
K_1	constant in Eq. (5)
n_*	constant exponent in Eq. (4)
Nu_b	Nusselt number $q_w b/[k(T_w - T_\infty)]$
Pr	Prandtl number, ν/a
q_w	local heat flux on the wall
r, φ, z	radial, tangential and axial coordinate
Re_φ	rotational Reynolds number $\omega b^2/\nu$
s	thickness of a slab or a disk
$T_w(t,r)$	instantaneous local temperature of the disk's surface
t	time
v_r, v_φ, v_z	radial, tangential and axial velocity components
x	dimensionless coordinate r/b
y	dimensionless coordinate $z/(s/2)$

Greek symbols

$\Delta T_t(t,r)$	instantaneous local temperature difference $T_w(t,r) - T_\infty$
$\Delta T(r)$	initial local temperature difference, $T_{w,i}(r) - T_\infty$
$\Theta(t,z)$	dimensionless temperature $(T - T_\infty)/(T_w(r,t) - T_\infty)$
$\theta(t,r,z)$	dimensionless temperature $(T - T_\infty)/c_{0*}$
ν	kinematic viscosity
ω	angular speed of rotation of the disk

Subscripts

i	initial conditions ($t = 0$)
w	wall ($z = 0$)
∞	infinity
1	disk's cylindrical surface
2	disk's flat surface

An advantage of solution (1) is its simplicity that can break once one has to take into consideration curvature of the surface at which the measurements are to be made [2].

An alternative to (1) is to use the solution for the unsteady heat transfer of a slab of a finite thickness $s/2$, where the back face is insulated [10], or a slab of a finite thickness s with identical heat transfer coefficients on both faces [1]

$$F_t(t) = \theta(t, y = 1),$$

$$\theta(t,y) = \sum_{m=1}^{\infty} E_m \cos(\mu_m y)\exp(-\mu_m^2 Fo), \qquad (2)$$

$$E_m = \frac{2\sin(\mu_m)}{\mu_m + \sin(\mu_m)\cos(\mu_m)}, \quad \cot(\mu_m) = \mu_m/Bi_2, \qquad (3)$$

where eigenvalues μ_m are defined by Eq. (3).

In particular, Eq. (2) was shown to agree well with the numerical solution of the unsteady conjugate heat transfer problem for a disk rotating in still air at laminar flow regime [9]. In agreement with [10], the authors [8,9] also suggested to use solution (2) and (3) to determine heat transfer coefficients basing on the instantaneous surface temperature measurements in view of the fact that (2) degenerates just to the first term of the Fourier series at $Fo \geqslant 0.3$. This reveals one of the important fundamental properties of the transient heat transfer such as an existence of the so-called regular regime of heating or cooling of a body when the temporal dependence of the local temperature in any location of the body follows a simple exponential function [1].

Strictly saying, both solutions (1) and (2) are valid only for the initially isothermal surfaces. In many industrial applications, the surface temperature varies at least in the streamwise direction. A remedy for such a situation was suggested via dividing the experimental surface on the rather narrow quasi-isothermal zones [3]. Within each zone, the surface temperature was assumed to be constant and undergoing a spatial step change at the boundary between the neighboring zones (instead of the monotonic temperature variation that takes place in practice). However, the validity of this approach is under question, since the experimental data [3] are themselves controversial. An attempt to find a quantitative estimate of the 2D heat conduction effects is made in [4].

All the investigations, performed in the present research, deal with a single rotating disk in still air (Fig. 1). This problem has itself an important meaning in view of the numerous industrial applications of rotating disks (turbomachinery, computer disk drives, electrochemistry etc.). Another reason is that the results presented in this paper are strongly based on the previously obtained outcomes of the transient heat transfer investigations of a single isothermal disk [8,9].

Thus, the objective of the present investigation consists in finding a quasi-conjugate heat transfer solution for a disk initially preheated non-uniformly, which includes (a) a self-similar solution of the transient laminar convective heat transfer, (b) a solution of the unsteady two-dimensional heat conduction problem with a non-uniform initial temperature distribution, (c) answer to the question

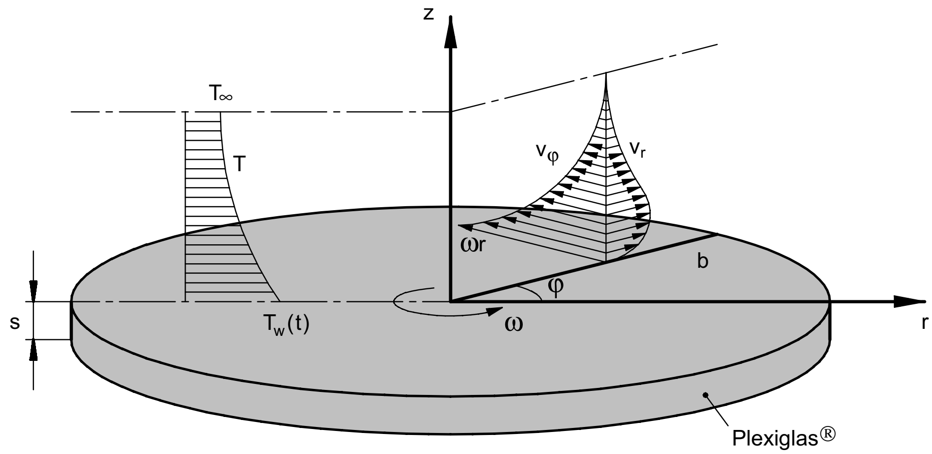

Fig. 1. Geometrical arrangement of a rotating disk in still air.

whether the shape of the initial temperature non-uniformity holds with time, and (d) validation of a transient technique for the experimental determination of the heat transfer coefficient basing on the regular heat transfer regime theory, which is free of the aforementioned restriction in the time of measurements.

The present research is aimed at clarifying the theoretical aspects of the transient experimental techniques of finding the heat transfer coefficients, i.e. validation of the different transient heat conduction solutions used in these techniques. Therefore we refer all the readers interested in technical details of the experimental technique discussed (e.g. elimination of the experimental noise etc.) to papers [2–7] and references.

2. Statement of the problem

A single disk rotating in still air is pre-heated in such a way that its surface temperature $T_{w,i}(r)$ follows a power-law distribution

$$\Delta T = c_{0*} x^{n_*}. \tag{4}$$

Boundary condition (4) has been widely used in modeling convective heat transfer in rotating-disk systems [11–15]. Eq. (4) has three major advantages. First, changing the exponent n_* from negative to positive values allows modeling a variety of the wall temperature distributions with T_w decreasing, constant or increasing in the streamwise direction. Second, boundary condition (4) allows obtaining an exact self-similar solution of the steady-state thermal boundary layer equation for laminar flow and very accurate approximate analytical solutions of the thermal boundary layer equation both for laminar and turbulent flows using an integral method [11–15]. Third, use of Eq. (4) engenders constant values of the heat transfer coefficients for laminar flow studied in the present paper that alleviates solution of the heat conduction problem.

While solving the heat conduction problem, one can easily discover two drawbacks of the boundary condition (4). First, it does not generally hold at $r = 0$, because it does not provide axial symmetry of the disk's temperature at this point. Second, choice of a thermally isolated outer cylindrical disk's surface invalidates Eq. (4) also at $r = b$. None of these restrictions is valid in solving the thermal boundary layer equation for an infinite-radius disk using boundary condition (4) [11–15]. Since such a solution is also an integral part of the present research, we still keep on using boundary condition (4) throughout the paper and intend also to figure out the magnitude of the possible numerical inaccuracies in the heat conduction solution caused by Eq. (4). More detailed insight into this particular feature of the obtained solution is presented in the section discussing the results.

Thus, it is assumed in the present research that distribution (4) exists at $t = 0$. Following [3,8,9], we consider a rotating disk ($b = 0.123$ m, $s = 0.01$ m), which at $t = 0$ starts cooling down to temperature T_∞ without additional input of heat.

The Nusselt number distribution for the steady-state conditions follows the power law [11–15]

$$Nu_b = K_1 Re_\varphi^{1/2}, \tag{5}$$

where the constant K_1 is tabulated in [11] depending on the values of the Prandtl number and n_*.

3. Self-similar solution of the transient laminar convective heat transfer problem

The unsteady thermal boundary layer equation for a disk is [8]

$$\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + v_z \frac{\partial T}{\partial z} = a \frac{\partial^2 T}{\partial z^2}. \tag{6}$$

Rewriting Eq. (6) using the non-dimensional temperature $\Theta(t, z)$, one obtains

$$\frac{\partial \Theta}{\partial t} + \frac{\Theta}{\Delta T_t} \frac{\partial \Delta T_t}{\partial t} + v_r \Theta \frac{1}{\Delta T_t} \frac{d\Delta T_t}{dr} + v_z \frac{\partial \Theta}{\partial z} = a \frac{\partial^2 \Theta}{\partial z^2}. \tag{7}$$

For the instantaneous heat transfer coefficient to be equal to its stationary value existing during the steady-state conditions before onset of the unsteady cooling/heating of the disk, ΔT_t should behave in such a way that

$$\Delta T_t(t, r) = \Delta T(r) F_t(t), \tag{8}$$

$$\frac{1}{\Delta T_t}\frac{\mathrm{d}\Delta T_t}{\mathrm{d}r} = \frac{1}{\Delta T}\frac{\mathrm{d}\Delta T}{\mathrm{d}r}, \tag{9}$$

with $F_t(t)$ to be a function of the only variable t. This is an indispensable condition, because the first two terms in Eq. (7) become negligible very fast [8], and namely the third term determines the effect of the radial disk's surface distribution on heat transfer.

In doing so Eq. (7) takes the following form [8]

$$\frac{\partial\Theta}{\partial t} + \frac{\Theta}{F_t}\frac{\partial F_t}{\partial t} + v_r\Theta\frac{1}{\Delta T}\frac{\mathrm{d}\Delta T}{\mathrm{d}r} + v_z\frac{\partial\Theta}{\partial z} = a\frac{\partial^2\Theta}{\partial z^2}. \tag{10}$$

In particular, in view of condition (4), Eq. (10) transforms to

$$\frac{\partial\Theta}{\partial t} + \frac{\Theta}{F_t}\frac{\partial F_t}{\partial t} + n_*\Theta\frac{v_r}{r} + v_z\frac{\partial\Theta}{\partial z} = a\frac{\partial^2\Theta}{\partial z^2}. \tag{11}$$

Employing self-similar functions and variables derived in [8] $F(\eta) = v_r r/t$, $G(\eta) = v_\varphi t/r$, $H(\eta) = v_z(t/v)^{1/2}$, $P(\eta) = -pt/(\rho v)$ and $\eta = z/(vt)^{1/2}$, one can reduce Eq. (11) to a self-similar form

$$\Theta'' = Pr[g_*\Theta + \Theta'(H - \eta/2) + n_* F\Theta],$$

$$g_* = \frac{t}{F_t}\frac{\mathrm{d}F_t}{\mathrm{d}t}, \tag{12}$$

where primes indicate derivatives with respect to η. The Nusselt number is calculated from the formula

$$Nu_b = K_1 Re_\varphi^{1/2}, \quad K_1 = -\frac{1}{\sqrt{\omega t}}\left(\frac{\mathrm{d}\Theta}{\mathrm{d}\eta}\right)_{\eta=0}. \tag{13}$$

Functions F, G, H and P are time-independent and can be found from the solution of the steady-state self-similar Navier–Stokes equations [8].

Comparisons of the self-similar solution results at $T_w = \text{const}$ (or $n_* = 0$) with the numerical simulations of this problem in the conjugate statement obtained using the commercial CFX-5 software [8] showed very good qualitative and quantitative agreement of both approaches. Therefore an extension of the self-similar analysis to the case of non-zero values of n_* is deemed justifiable and capable of producing plausible results.

4. Solution of the unsteady two-dimensional problem of heat conduction in a disk

Equation of unsteady 2D heat conduction in the disk and boundary conditions are [1]

$$\frac{\partial\theta}{\partial Fo} = \frac{1}{H^2}\left(\frac{\partial^2\theta}{\partial x^2} + \frac{1}{x}\frac{\partial\theta}{\partial x}\right) + \frac{\partial^2\theta}{\partial y^2}, \tag{14}$$

$$Fo = 0: \quad \theta = x^{n_*}, \tag{15}$$

$$x = 0: \quad \frac{\partial\theta}{\partial x} = 0, \quad x = 1: \quad \frac{\partial\theta}{\partial x} = -Bi_1\theta, \tag{16}$$

$$y = 0: \quad \frac{\partial\theta}{\partial y} = 0, \quad y = 1: \quad \frac{\partial\theta}{\partial y} = -Bi_2\theta. \tag{17}$$

This system is solved using the method of separation of variables [1]. The resulting solution has the following form:

$$\theta(Fo, x, y) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} D_n E_m J_0(\mu_{xn}x)\cos(\mu_{ym}y) \times \exp[-(\mu_{xn}^2/H^2 + \mu_{ym}^2)Fo], \tag{18}$$

$$D_n = \frac{{}_1F_2(1 + n_*/2; 1, 2 + n_*/2; -\mu_{xn}^2/4)/(2 + n_*)}{0.5[J_0^2(\mu_{xn}) + J_1^2(\mu_{xn})]},$$

$$\frac{J_1(\mu_{xn})}{J_0(\mu_{xn})} = \frac{Bi_1}{\mu_{xn}}. \tag{19}$$

Here, ${}_1F_2$ is a hypergeometric function of the argument $-\mu_{xn}^2/4$ [16,17]. Constant E_m and eigenvalues μ_{ym} are given by Eqs. (3). At $n_* = 0$ the numerator of Eq. (19) is equal to $J_1(\mu_{xn})/\mu_{xn}$, and Eq. (18) coincides with the solution presented in [1]. Obviously, neglecting radial heat conduction effects one can reduce Eq. (18) to Eq. (2).

Using a thin-slab approximation, one can neglect temperature variation inside the disk in the y-direction and substitute the last term in Eq. (14) with the so-called source term $-\theta Bi_2$. In doing so, the final solution of Eq. (14) reduces to

$$\theta(Fo, x) = \sum_{n=1}^{\infty} D_n J_0(\mu_{xn}x)\exp[-(\mu_{xn}^2/H^2 + Bi_2)Fo]. \tag{20}$$

5. Analysis of the solutions for unsteady heat conduction in a disk

5.1. Convective heat transfer

In the present research we study effects of the non-uniform initial temperature distribution for $n_* = -1$, -0.5, -0.25, 0, 0.5, 1 and 2 at $Pr = 0.71$. For the steady-state conditions this results in values of the constant K_1 in Eq. (5) equal to $K_1 = 0.1893$, 0.2624, 0.2952, 0.3259, 0.3818, 0.4319 and 0.5185, respectively.

The basic modeling variant deals with a disk made of Plexiglas® with its low heat conductivity. The physical properties of Plexiglas® are [3] $k_w = 0.19$ W/(m^2 K), $a_w = 1.086 \times 10^{-7}$ m^2/s and of air [1] $k = 0.02624$ W/(m^2 K), $a = 2.216 \times 10^{-5}$ m^2/s, $Pr = 0.71$.

For comparison purposes, we have also computed unsteady cooling of a disk made of aluminum, whose physical properties are [18] $k_w = 204$ W/(m^2 K), $a_w = 0.842 \times 10^{-4}$ m^2/s.

Exactly as in [8,9], we hold the value of $Bi_2 = 0.395$ constant throughout the numerical computations. Once K_1 varies with n_*, the condition $Bi_2 = \text{const}$ means a different value of Re_φ for every particular n_* provided that $Re_\varphi = 5.35 \times 10^4$ at $n_* = 0$.

In works [8,9] the author mentioned that the constant K_1 attains its stationary values for $n_* = 0$ already at $\omega t \approx 1000$ or $t \approx 19$ s with F_t being equal to 0.876. In this case the deviation of K_1 from its stationary value makes 0.14% that is obviously a too strict criterion. Assuming a standard 1% deviation being sufficient, one can obtain the stationary value of K_1 already at $\omega t \approx 130$, $t \approx 2.5$ s with F_t being equal to ≈0.96. Basing on the very same 1% deviation, the constant K_1 reaches its stationary value at $\omega t \approx 40$ for $n_* = 2$ and at $\omega t \approx 300$ for $n_* = -1$. In both cases $t \approx 2$ s, while the Reynolds number Re_φ and angular speed of rotation ω decrease with increasing n_* to keep the Biot number constant at $Bi_2 = 0.395$. For these small process times Eqs. (2) and (3) coincide and predict values of F_t equal to $F_t \approx 0.96$.

Assuming that $Re_\varphi = 5.35 \times 10^4$ is invariant, K_1 anyway becomes stationary very quickly: at $t \approx 5.5$ s for $n_* = -1$ and at $t \approx 0.7$ s for $n_* = 2$.

In order to render the temporal dependencies of all the functions in the present paper the most general appearance, a non-dimensional time in the form of the Fourier number was used. The interrelation between the real and non-dimensional time is: for the Plexiglas® disk $t = 230.17Fo$, for the aluminium disk $t = 0.297Fo$.

5.2. Unsteady temperature distributions in an isothermal disk at $n_* = 0$

Detailed results for this case are documented in [8,9]. Here, we present some new findings in comparison with what was found in [8,9]. Variation of the non-dimensional disk surface temperature F_t with time is shown in Fig. 2. One-dimensional (2) and two-dimensional (18) solutions for a slab of a finite thickness coincide over the whole range of the variation of the Fourier number. Solution for a slab of an infinite thickness (1) agrees well with Eqs. (2) and (18) only up to a certain limiting value of Fo, which can be estimated as $Fo = 0.456$, 0.608 and 0.790 when the divergence of Eqs. (1) and (2) is set as 1%, 2.5% and 5%, respectively. Having passed this threshold value of the argument Fo, data from Eq. (1) exceed more and more noticeably solutions (2) and (18). From the physical point of view this means that a finite-thickness disk cools down more and more rapidly than an infinite-thickness one. Solution (20) for a thin disk deviates from Eq. (18) quite noticeably at $Fo < 2$ an $Fo > 4$. This means that the 0.01 m thick disk considered in the present paper as a basic geometry is insufficiently thin for the solution (20) to be valid. Results of additional computations showed that for numerical data from Eqs. (18) and (20) to coincide the disk should be 0.001 m thick. Thus, solution (20) has no use for the present research and will not be discussed further in this paper.

5.3. Unsteady temperature distributions in a non-isothermal disk

At the beginning we will discuss the results obtained for the disk made of Plexiglas®. The results obtained basing on solution (18) differ for the cases where n_* is moderately or strongly different from zero. Cases with $n_* = -1$ and 2 illustrated in Figs. 3 and 4 should be classified as those strongly different from zero. During the cooling process, curves of the normalized wall temperature $\theta_w/\theta_w(x=1)$ seem to repeat the initial power-law distributions (4) for the most part of the disk. Plots of $\theta_w/\theta_w(x=1)$ at $n_* = -0.25$, -0.5, 0.5 and 1 (omitted here to save space) behave in the same way. There are two visible specific features of the plots of $\theta_w/\theta_w(x=1)$ worth mentioning here.

First, Eq. (4) does not agree in general with boundary conditions (16) and (17). Therefore, solution (18) adjusts itself to Eqs. (16) and (17) as Fo increases, thus creating instantaneous distributions of $\theta_w/\theta_w(x=1)$ distorted in the neighborhood of the points $x = 0$ and $x = 1$ as compared with Eq. (4). Hence, the temporal behavior of the heat transfer coefficient $h_{2,t}$ should be studied within a narrower region, say, at $x = 0.2$–0.9.

Second, at small Fourier numbers the aforementioned mathematical contradiction of Eqs. (4) and (16), (17) engenders oscillations of the temperature profiles visible in Figs. 4 and 5. This is a modest price to be paid for the

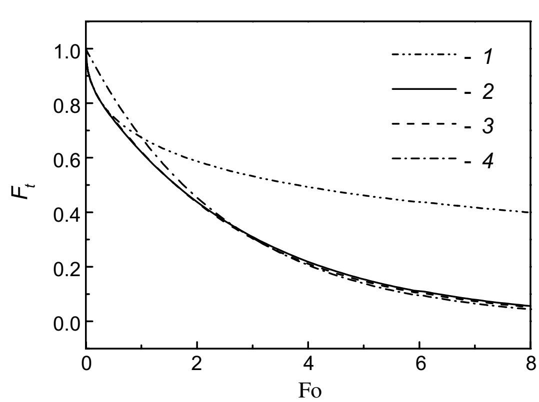

Fig. 2. Variation of F_t versus Fo at $n_* = 0$. 1 – Eq. (1); 2 – Eq. (2); 3 – Eq. (18); 4 – Eq. (20).

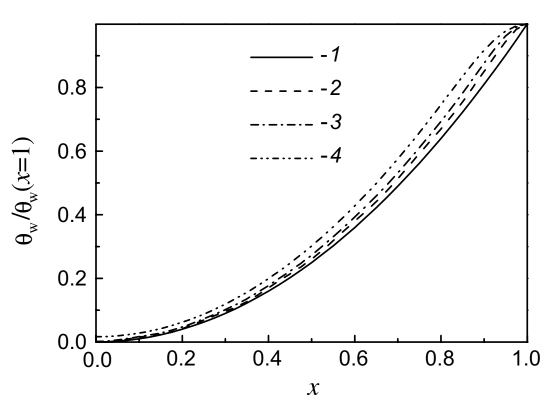

Fig. 3. Variation of $\theta_w/\theta_w(x=1) = \theta(Fo, x, y=1)/\theta(Fo, x=1, y=1)$ according to Eq. (18) versus x at $n_* = 2$. 1 – Eq. (4); 2 – $Fo = 0.217$; 3 – $Fo = 0.652$; 5 – $Fo = 2.172$.

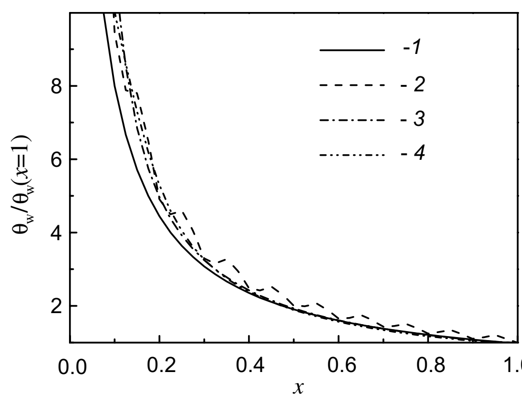

Fig. 4. Variation of $\theta_w/\theta_w(x=1) = \theta(Fo, x, y=1)/\theta(Fo, x=1, y=1)$ according to Eq. (18) versus x at $n_* = -1$. 1 – Eq. (4); 2 – $Fo = 0.00434$; 3 – $Fo = 0.652$; 5 – $Fo = 2.172$.

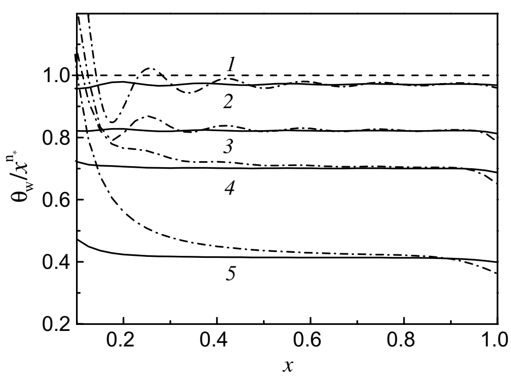

Fig. 5. Variation of $\theta_w/x^{n_*} = \theta(Fo, x, y=1)/x^{n_*}$ according to Eq. (18) versus x at $n_* = 2$ (dash-dotted lines) and $n_* = 0.5$ (solid lines) for different Fo. 1 – Eq. (4); 2 – $Fo = 0.00434$; 3 – $Fo = 0.217$; 4 – $Fo = 0.652$; 5 – $Fo = 2.172$.

possibility to operate with the boundary condition (4) necessitated in Section 2.

An opportunity to discern the differences in the cases with moderate and strong deviations of n_* from zero is provided in Fig. 5 where the instantaneous surface temperature profiles θ_w are plotted as divided by x^{n_*}. The enlarged scale of Fig. 5 shows that for the initial radial surface temperature distributions (4) moderately different from the isothermal case (e.g. $n_* = 0.5$) the non-stationary plots of θ_w/x^{n_*} look like horizontal straight lines over the region $x = 0.2$–0.95 (and even the initial oscillations at $Fo = 0.00434$ are almost negligible). This in turn means that conditions (8) and (9), which are necessary for the transient experiment to be valid, hold at $x = 0.2$–0.95 for the whole duration of the cooling process. When the temperature distributions (4) strongly deviate from the isothermal case (e.g. $n_* = 2$), the plots of θ_w/x^{n_*} at relatively small values of Fo (e.g. still at $Fo = 0.217$) oscillate around some horizontal lines. However, following the further increase in Fo the curves of θ_w/x^{n_*} become more noticeably inclined towards the outer edge of the disk (i.e. towards the value $x = 1$). The reason lies in the redistribution of heat owing to the radial heat conduction from the more heated outer part of the disk into its less heated inner part. Eventually, one obtains a distorted (as compared with Eq. (4)) distribution of θ_w/x^{n_*} at high values of Fo, which does not comply in full with conditions (8) and (9). This phenomenon is more noticeable for positive values of n_* than for negative ones, because at positive of n_* the total amount of heat accumulated by the hotter part of this disk is larger (because the hotter part of the disk is volumetrically larger in this case).

5.4. Transient values of the heat transfer coefficient based on the infinite-slab solution

However, the target quantity in the present investigation is the heat transfer coefficient rather than the surface temperature. Values of the heat transfer coefficient computed with the use of Eqs. (1) and (2) for an isothermal disk at $n_* = 0$ are presented in Fig. 6. Assuming Eq. (2) to be a true solution for the unsteady heat transfer and specifying a constant value of h_2 in the boundary condition (17), we have equated Eqs. (1) and (2) and computed "transient" values of the heat transfer coefficient $h_{2,t}$ from Eq. (1) depending on the Fourier number. In doing so we have imitated the conditions that take place during transient experiments, with Eq. (2) playing a role of the "experimental" data source. The data obtained and plotted in Fig. 6 as curve 1 testify that experimental technique based on the use of Eq. (1) can produce valid experimental data for $h_{2,t}$ again up to the certain limiting value of Fo. It can be estimated as $Fo = 0.3$, 0.391 and 0.487 in order for the deviation of $h_{2,t}$ from h_2 not to exceed 1%, 2.5% and 5%, respectively. These threshold values are lower than those obtained in subsection 5.2 at the comparison of values F_t from Eqs. (1) and (2). The explanation lies in the fact that an increase in $h_{2,t}$ leads to a correspondent increase in the function $\exp(\gamma^2)$ and a decrease in the function $\mathrm{erfc}(\gamma)$, which form a product in Eq. (1) and thus mutually compensate in part the deviation of $h_{2,t}$ from the pre-determined value h_2. Thus, basing on the 1%-restriction in the error of $h_{2,t}$, the limiting measurement time should be less

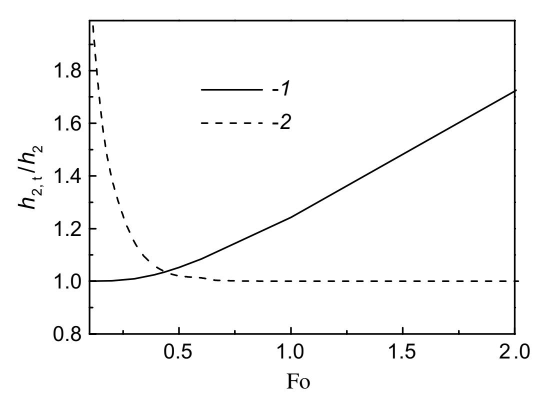

Fig. 6. Variation of the ratio h/h_2 versus Fo at $n_* = 0$. 1 – Eq. (1); 2 – Eq. (2).

than $Fo = 0.3$ or 69 s that agree rather with the classical value $Fo = 1/4$ [6] than with the revised suggestion $Fo = 1$ [7].

The data for the heat transfer coefficient $h_{2,t}$, obtained for all studied input values of n_* from -1 to 2 using the infinite-slab solution and averaged over the region $x = 0.2$–0.9 to avoid the oscillations of the obtained solution at low Fo, agree at the maximal discrepancy of 1% with line 1 in Fig. 6. Thus the fundamental conclusion from this finding is that the infinite-slab approach (1) is still valid as an experimental transient technique also for the cases with strong initial radial (i.e. streamwise) temperature gradients in the surface under investigation. Of course, this conclusion is justifiable for the disks made of Plexiglas®.

5.5. Transient values of the heat transfer coefficient based on the regular heat transfer regime theory

An alternative technique of determining the heat transfer coefficient from the experimentally measured instantaneous distributions of the surface temperature is based on the theory of the regular heat transfer regime [1]. The key statement of this theory is that with the increasing Fourier number the series solutions (2) and (18) degenerate just to their first terms. Taking a derivative of logarithm of the first term of (18) with respect to time, one can obtain

$$-\frac{\partial\theta(Fo,x,1)}{\partial t} = (\mu_{x1}^2/H^2 + \mu_{y1}^2)\frac{a}{(s/2)^2} = m, \tag{21}$$

$$-\frac{\partial\theta(Fo,x,1)}{\partial t} = \mu_{y1}^2\frac{a}{(s/2)^2} = m. \tag{22}$$

Solution (22) for the disk, whose outer rim is thermally insulated to avoid radial conduction effects in this location ($\mu_{x1} = 0$), is more convenient and will be used in the further derivations. The regular regime of heat transfer takes place when the experimentally measured curve of the function $-\partial\theta(Fo,x,1)/\partial t$ becomes constant m. Having experimentally found m and keeping in mind Eq. (3), one can easily deduce

$$\mu_{y1} = 0.5s\sqrt{m/a}, \quad h_{2,t} = \frac{k\sqrt{m/a}}{\cot(0.5s\sqrt{m/a})}. \tag{23}$$

Obviously, Eqs. (22) and (23) can be obtained from solution (2), (3) that disregards radial conduction effects. Assuming again that Eq. (2) is a true solution for the unsteady heat transfer at a constant value of h_2 in the boundary condition (17), we have computed instantaneous values of m and "transient" values of the heat transfer coefficient $h_{2,t}$ depending on Fo. Values of $h_{2,t}$ presented in Fig. 6 decrease quite rapidly. Already at $Fo = 0.543$ errors in determination of $h_{2,t}$ are equal to 1% and become negligible with the further increase in the Fourier number.

The advantage of approach (23) consists in the fact that the regular heat transfer regime, once it is already established, imposes no restriction on the duration of measurements within the practical timescale providing visible differences between the surface and ambient temperatures. Within these time limits the heat transfer coefficient $h_{2,t}$ remains constant.

At moderate radial surface temperature gradients, i.e. at n_* from -0.5 to 0.5, and for radial locations at $x = 0.25$–0.9, solution (23) as a basis for the experimental technique is still valid. The curves of the radial distributions of the heat transfer coefficient $h_{2,t}$ obtained using the regular heat transfer regime theory and plotted in Fig. 7 can serve as an evidence of this conclusion. At n_* equal to -1 and 1 the region of the validity of Eq. (23) narrows down to $x = 0.45$–0.85, while at $n_* = 2$ approach (23) cannot be recommended at all. Thus the conclusion is that the regular heat transfer regime theory has restrictions attributed to the radial heat conduction effects at high values of n_*, which distort the initial temperature distribution (4) and finally the computed radial curves for $h_{2,t}$ at high Fo. A remedy may consist in the use of the disks (or, in general, the objects of the experimental study) made of the materials with lower thermal conductivity in comparison with that of Plexiglas®.

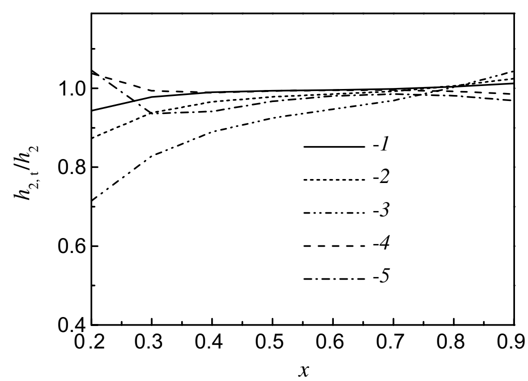

Fig. 7. Variation of the ratio h/h_2 versus x at $Fo = 5$. 1 – $n_* = 0.5$; 2 – $n_* = 1$; 3 – $n_* = 2$; 4 – $n_* = -0.5$; 4 – $n_* = -1$.

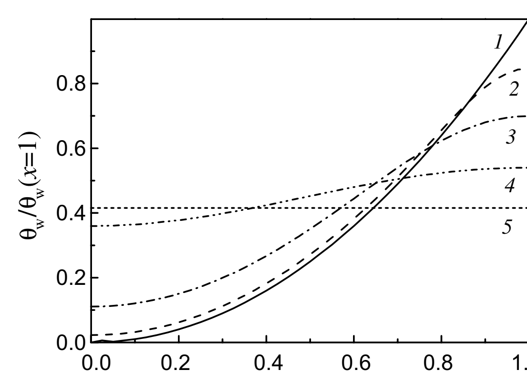

Fig. 8. Variation of $\theta_w/\theta_w(x=1) = \theta(Fo,x,y=1)/\theta(Fo,x=1,y=1)$ according to Eq. (18) versus x at $n_* = 2$ for aluminium. 1 – Eq. (4); 2 – $Fo = 3.367$ ($t = 1$ s); 3 – $Fo = 15.835$ ($t = 5$ s); 5 – $Fo = 67.34$ ($t = 20$ s); 5 – $Fo = 505.05$ ($t = 150$ s).

5.6. Disk made of aluminium

In order to further illustrate the effect of the disk's material, we have modeled unsteady cooling of a disk made of aluminum. From the very beginning of the cooling process even technique (1) is inapplicable, because even at $t = 1$ s the distribution of $\theta_w/\theta_w(x = 1)$ becomes radically distorted in comparison with (4) owing to the radial heat conduction (see Fig. 8), so that one obtains very high positive values of $h_{2,t}$ at $x > 0.85$ and very high negative values at $x < 0.8$. Very rapidly the disk at the initial value $n_* = 2$ becomes isothermal.

6. Conclusions

As the important results of the research fulfilled we would like to mention the following:

(1) A self-similar solution of the transient laminar convective heat transfer for the initially non-isothermal disk. Similarly to what was obtained earlier for an isothermal disk, the values of the heat transfer coefficient reached the steady-state values very rapidly (in 2–5.5 s).
(2) An analytical solution (18) of the unsteady two-dimensional heat conduction problem at the non-uniform initial temperature distribution (4).
(3) Formulation of conditions (8) and (9) necessary for the shape of the initial temperature non-uniformity to hold with time. These conditions are fulfilled in full for the disk made of Plexiglas® in the sense of the applicability of the infinite-slab approach (1), which thus may be used as an experimental technique for determining the heat transfer coefficients $h_{2,t}$ at all the studied values of n_* from -1 to 2.
(4) Validation of a transient technique for the experimental determination of $h_{2,t}$ basing on the regular heat transfer regime theory. This technique is valid in full at n_* varying from -0.5 to 0.5, partially valid for n_* equal to -1 and 1, and cannot be recommended for $n_* \geqslant 2$.

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